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Stability by Linear Processes

Problem: To find a quick way to determine if the origin O in the complex plane **C** within the image of Q under the multilinear function $p(j\omega,q) = f(q) + g(q)j$ for fixed ω.

Notation:

Parameter Space R^n	**C**
1. $A_1, A_2, \ldots$ in $Q \subset R^n$ are preimages of $A_1', A_2', \ldots$ under $p(j\omega,q)$	$A_1', A_2', \ldots$ points in **C**
2. l_1 edge and $l_2, \ldots l_n$ line segments where $l_1 \perp l_2 \ldots \perp l_n$	$l_1', l_2', \ldots, l_n'$ line segments in **C** which are images of line segments $l_1, l_2, \ldots, l_n$ respectively from Q.

Without loss of generality, assume that the point $(q_1^-, q_2^-, \ldots, q_n^-)$ and the point $(q_1^+, q_2^-, \ldots, q_n^-)$ are the endpoints of the edge l_1 and the point $A_1 \in l_1$ would be $(q_1, q_2^-, \ldots, q_n^-)$ then the point $(q_1, q_2^-, \ldots, q_n^-)$ and the point $(q_1, q_2^+, q_3^- \ldots, q_n^-)$ are the endpoint of l_2 and the point $A_2 \in l_2$ is $(q_1, q_2, q_3^- \ldots, q_n^-)$. So the point $(q_1, \ldots, q_i^-, q_{i+1}^-, \ldots, q_n^-)$ and the point $(q_1, \ldots, q_i^+, q_{i+1}^-, \ldots, q_n^-)$ are the endpoints of l_i and the point $A_i \in l_i$ is $(q_1, \ldots, q_i, q_{i+1}^-, \ldots, q_n^-)$ and so forth.

Algorithm:

1. Map any edge, l_1, of $Q \subset R^n$ to the line segment l_1' in **C**.

2. If the line through O and A_1' is not perpendicular to l_1' then we are finished.

3. Determine the point A_1' on l_1' that is closest to O, if the line through O and A_1' is perpendicular to l_1.

   a) Once $(q_1^-, q_2^-, \ldots, q_n^-)$ and $(q_1^+, q_2^-, \ldots, q_n^-)$ is mapped to **C**, say $f(q_1^\pm) = f(q_1^\pm, q_2^-, \ldots, q_n^-)$ and $g(q_1^\pm) = g(q_1^\pm, q_2^-, \ldots, q_n^-)$

   b) Then the slope

$$m = \frac{g(q_1^-) - g(q_1^+)}{f(q_1^-) - f(q_1^+)}$$

   is calculated.

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c) The closest point A_1' (x,y) to O in $\mathbb{C}$ is

$$x = \frac{mf(q_1^-) - g(q_1^-)]}{(m + 1/m)}$$

$$y = -x/m$$

d) Consider the preimage A_1 of A_1'. It is easy to determine the point A_1 lies on the edge l_1 on Q by linearity.

4. Construct the line $l_2 \perp l_1$ through A_1 in Q along with ith axis.

   a) Since all of the q's are fixed except q_i then the ith coordinate of A_1 is founded by

$$q_i = \frac{\text{x - all linear factors not containing } q_i}{\text{all linear factor containing } q_i}$$

5. Repeat the process for the lines l_i, $i = 3,\ldots,n$ as was done for l_2.

The speed of the algorithm can be determined by examining the best/worst case scenario. The best case is that the first edge tried does not have a perpendicular through O which means we exit the algorithm. The worst case is that all of the n-1 directional edges have perpendiculars. This requires n iterations. Therefore the average case requires (n+1)/2 iterations. One iteration includes the following calculations:

1. f and g for an edge
2. the slope of the line segment in $\mathbb{C}$
3. the point (x,y) which is perpendicular to the line through O
4. determining if (x,y) lies within the line segment
5. mapping (x,y) to the preimage

Conjecture 1: $p(j\omega,q) \in$ Hurwitz $\forall\ q \in Q$ Iff for some $i \in \{1,...,n\}$ the line through O is not perpendicular to l_i'

Conjecture 2: $p(j\omega,q) \notin$ Hurwitz $\forall\ q \in Q$ Iff $d_i = \|A_i' - O\|$, $i = 1,...,n$ then $d_i > d_{i+1}$ for each $i = 1,...,n-1$.

To prove the above conjectures we are considering the following direction.

Lemma 1:

If for all edges of Q map to line segment in $\mathbb{C}$ no perpendicular to these segments pass through O, then all line segments l_i', $i=2,...,n$ do not have a perpendicular through O.

Lemma 2:

Given any line segment l_j in Q parallel jth axis consider all line segments l_{j+1}^k , $k=1,2,...,n-1$ such that l_{j+1}^k along with $(j+1)$ axis then $l_{j+1}^{k\prime}$ are all on the same side of l_j'.

Lemma 1 has already been proven.

Example:

$f(q_1,q_2,q_3,q_4) = q_1\ q_2\ q_3\ q_4 + q_1 + q_3 + q_2\ q_3 + 1$
$g(q_1,q_2,q_3,q_4) = q_1\ q_2\ q_3 + 2\ q_1 + q_2 + q_3 + q_4$

1st pass
endpoints (-2,-2,-2,-2) (-2,-2,-2, 2)
image in $\mathbb{C}$ (17,-18) (-15,-14)
$\perp$ point (-1.953846,-15.63077) inside line segment

2nd pass
(-2,-2,-2,.3692307) (2,-2,-2,.3692307)
(-1.953846,-15.63077) (7.953846,8.36923)
$\perp$ point (3.843794,-1.586797) inside line segment

3rd pass
(.340662,-2,-2,.3692307) (.340662,-2,2,.3692307)
(3.843794,-1.586797) (-1.16247,-.3120933)
$\perp$ point inside (-.1454044 -.5710602) inside line segment

4th pass
(.340662,-2,1.187366,.3692307) (.340662,2,1.187366,.3692307)
(-.1454044,-.5710603) (5.20146,5.046901)
$\perp$ point inside (.2088863,-.1988064) inside line segment

Then $0 \in \mathrm{Im}\{p(s,Q)\}$

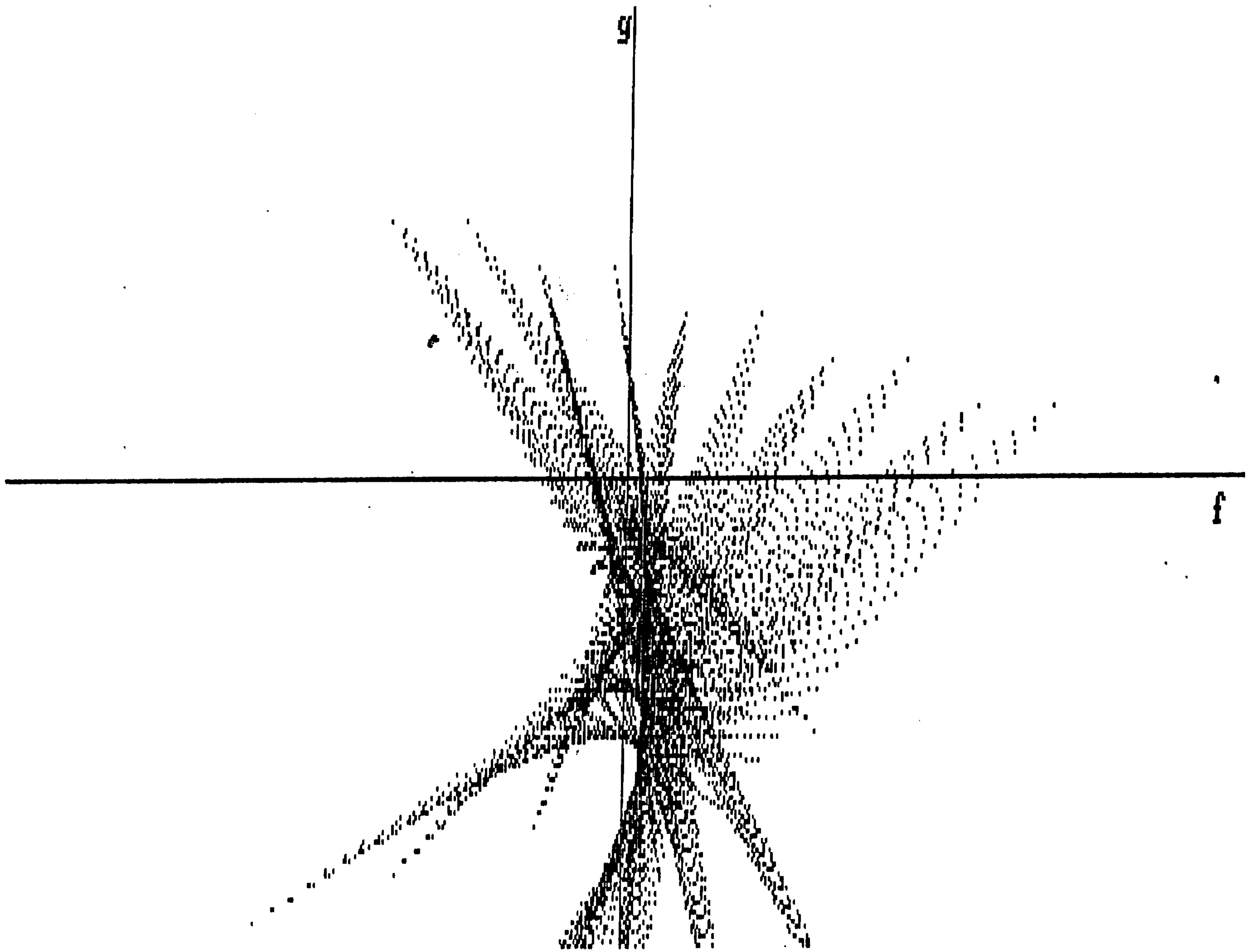

Figure 1a

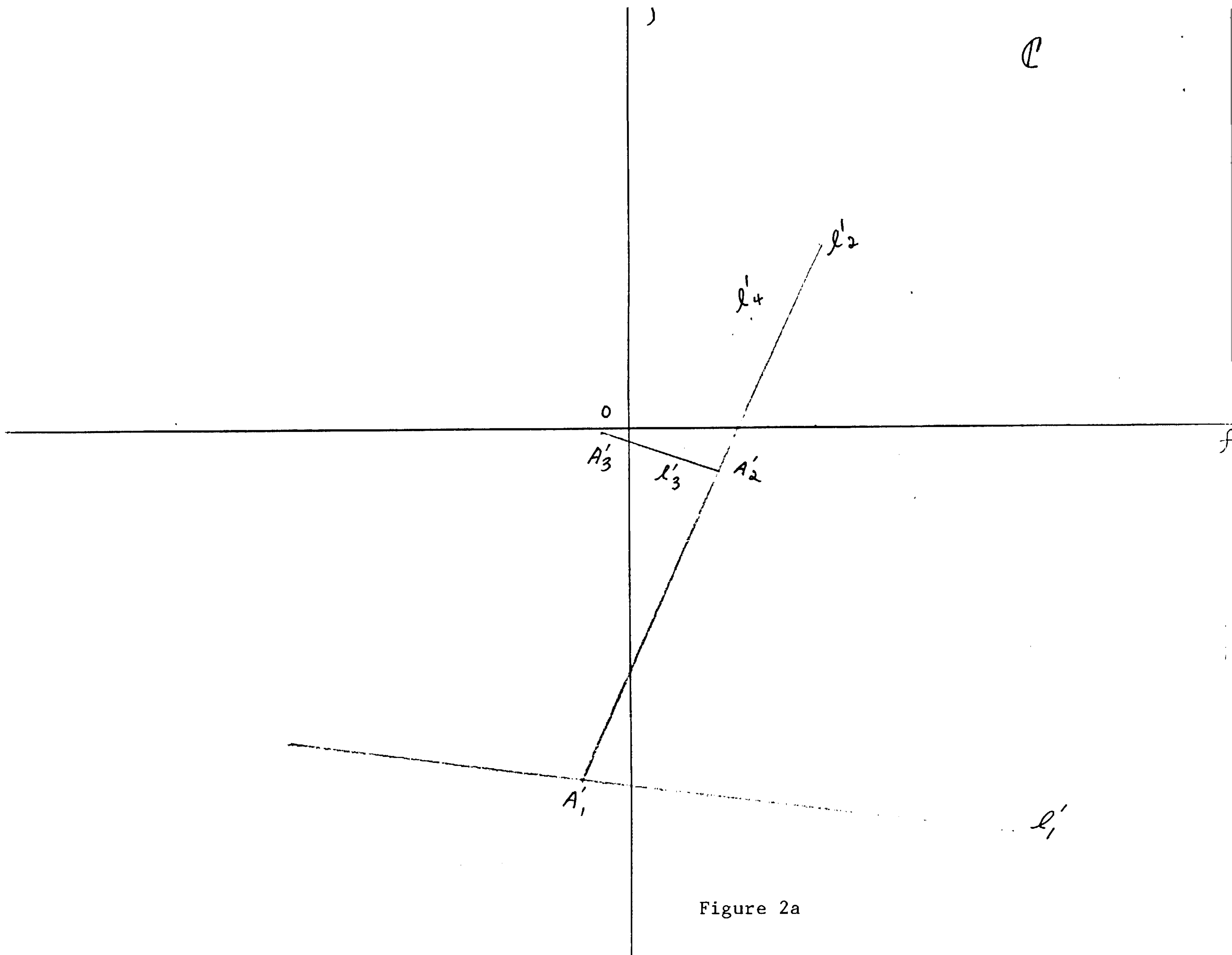

Figure 2a

$$f(q_1,q_2,q_3,q_4) = q_1q_2q_3q_4 + q_1 + q_2 + q_3 + q_2q_3 - 2q_1q_2q_3 - 10$$
$$g(q_1,q_2,q_3,q_4) = -q_1q_2q_3 + 2q_1 + q_2 + q_3 + q_4 - q_2q_3 - q_3q_4 + 2$$

1st pass
endpoints (-2,-2,-2,-2) (-2,-2,-2,2)
image in $\mathbb{C}$ (20,-8) (-12,4)
$\perp$ point (-.1643836,-.4383562) inside line segment

2nd pass
(-2,-2,-2,.5205479) (2,-2,-2,.5205479)
(-.1643834,-.4383562) (-19.83562,-8.438356)
$\perp$ point (.1296431,-.31878) outside line segment

Then $0 \notin \mathrm{Im}\{p(s,Q)\}$

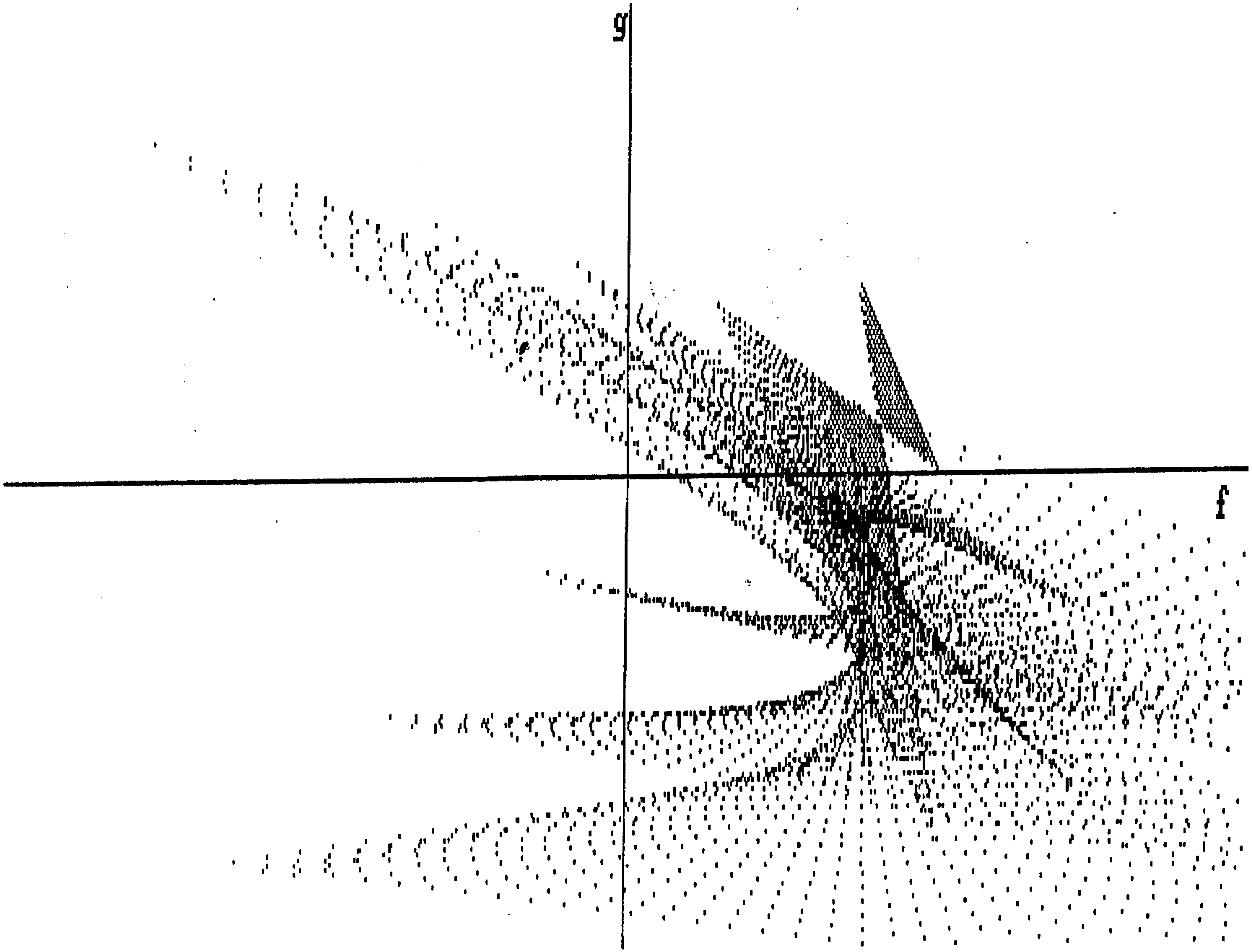

Figure 1b

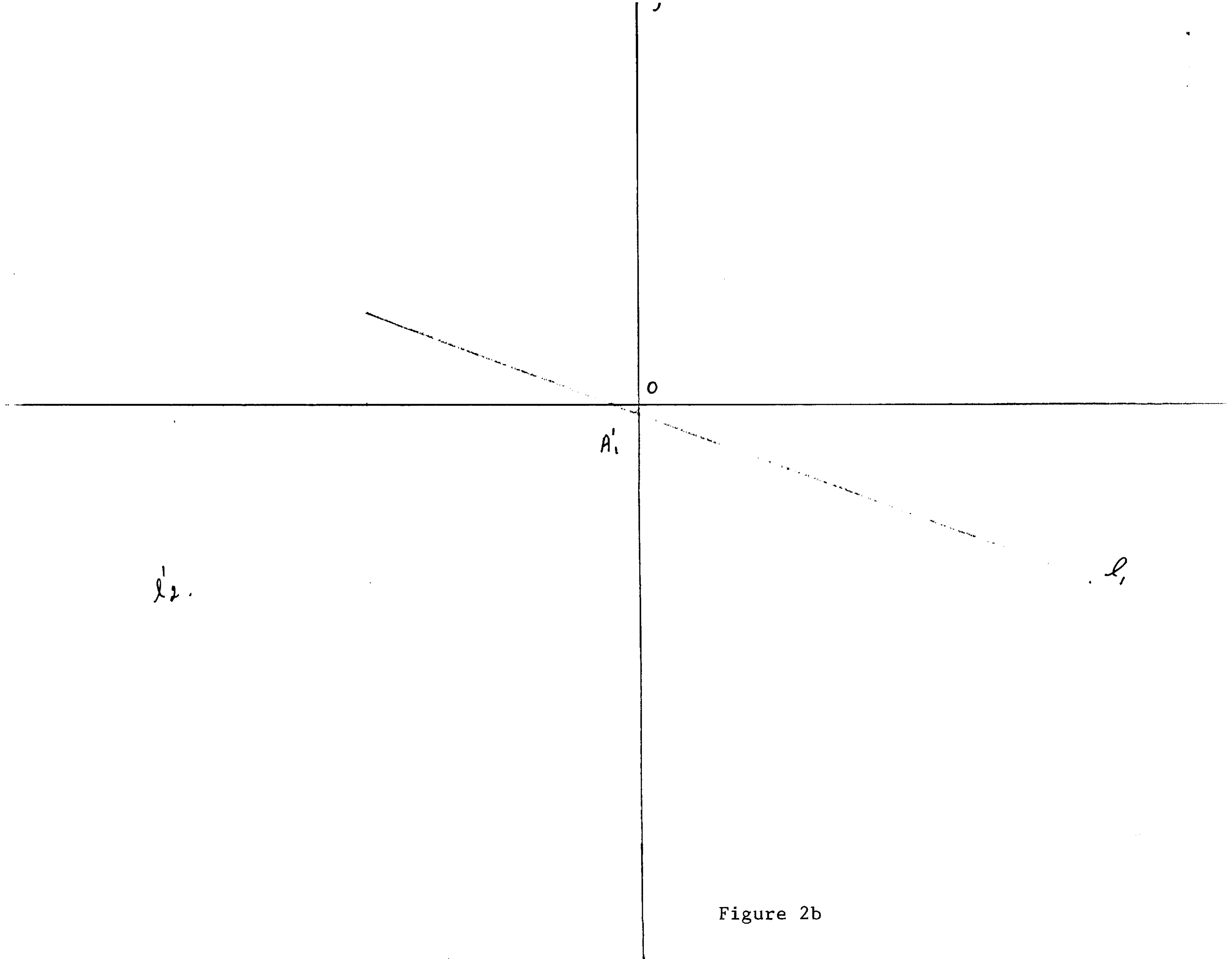

Figure 2b